

# Could $Z_b(10610)$ be a $B^*\bar{B}$ molecular state?

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Assuming the newly observed structure  $Z_b(10610)$  as a bottomonium-like molecular state  $B^*\bar{B}$ , we calculate its mass in the framework of QCD sum rules. The numerical result is  $10.54 \pm 0.22$  GeV for  $B^*\bar{B}$ , which coincide with the mass of  $Z_b(10610)$ . This consolidates the statement made by Belle Collaboration that the  $Z_b(10610)$  resonance could be a  $B^*\bar{B}$  molecular state.

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## I. INTRODUCTION

Very recently, Belle Collaboration observed two narrow structures  $Z_b(10610)$  and  $Z_b(10650)$  in the  $\pi^\pm \Upsilon(nS)$  ( $n = 1, 2, 3$ ) and  $\pi^\pm h_b(mP)$  ( $m = 1, 2$ ) mass spectra that are produced in association with a single charged pion in  $\Upsilon(5S)$  decays [1]. The measured masses of the two structures are  $10608.4 \pm 2.0$  MeV and  $10653.2 \pm 1.5$  MeV, respectively. Experimental analysis favors quantum numbers of  $J^P = 1^+$  for both states. As  $Z_b(10610)$  and  $Z_b(10650)$  are charged, they can not be simple  $b\bar{b}$  combinations. The measured masses of these two new states exceed by only a few MeV the thresholds for the open beauty channels  $B^*\bar{B}$  and  $B^*\bar{B}^*$ . They could be interpreted as molecular states and determined by the strong interaction dynamics of  $B^*\bar{B}$  and  $B^*\bar{B}^*$  meson pairs [1–3].

The concepts of molecular states were put forward long ago in [4]. Some of the exotic X, Y, and Z resonances have been described as possible charmonium-like molecular candidates in the literatures since their masses are very close to the meson-meson thresholds. Explicitly, it is interpreted  $Z^+(4430)$  as a  $D^*\bar{D}_1$  molecular state [5],  $Y(3930)$  as a  $D^*\bar{D}^*$  molecular state [6–8],  $Y(4140)$  as a  $D_s^*\bar{D}_s^*$  molecular state [7, 9],  $Y(4260)$  as a  $\chi_{c\rho}^0$  [10] or an  $\omega\chi_{c1}$  molecular state [11],  $X(4350)$  as a  $D_s^*D_{s0}^*$  molecular state [12, 13],  $Y(4274)$  as a  $D_sD_{s0}(2317)$  molecular state [14] etc.. If molecular states can be confirmed, QCD will be further testified and then one will understand QCD low-energy behaviors more deeply.

The newly observed  $Z_b$  resonances may open a new window to study molecular states in the bottomonium-like family. Therefore, it is interesting to investigate whether they could be bottomonium-like molecular candidates. The quantitative description of their properties like masses are helpful for understanding their structures. Unfortunately, quarks are confined inside hadrons in the real world, and the strong interaction dynamics of these states are governed by nonperturbative QCD effect completely. In this work, by assuming  $Z_b(10610)$  as a  $B^*\bar{B}$  molecular state, we calculate the mass of this resonance in the framework of QCD sum rule (QCDSR) method [15], which is a nonperturbative formulation firmly rooted in QCD basic theory and has been used to study some charmonium-like molecular states [16–18]. It is not so straightforward from the meson-meson configuration of fields to construct a  $B^*\bar{B}^*$  current with a quantum number of  $J^P = 1^+$ . The  $Z_b(10650)$  will not be discussed here. Our final numerical result  $10.54 \pm 0.22$  GeV for  $B^*\bar{B}$  agrees well with the experimental data of  $Z_b(10610)$ , while the masses of the  $J^P = 1^+$   $b\bar{b}q\bar{q}$  tetraquark states were found to be around  $10.1 \sim 10.3$  GeV in QCDSR [19, 20], which are lower than the measured values of  $Z_b$  states. The present work thus favors that the  $Z_b(10610)$  resonance could be a  $B^*\bar{B}$  molecular state rather than a  $b\bar{b}q\bar{q}$  tetraquark state.

The rest of the paper is organized as three parts. We discuss QCD sum rules for the molecular state in Sec. II, where the phenomenological representation and the operator product expansion (OPE) contribution up to dimension six operators for the two-point correlator are derived. The numerical analysis is made in Sec. III. The mass of the  $B^*\bar{B}$  molecular state is extracted out and found to coincide with the experimental value of  $Z_b(10610)$  resonance. The Sec. IV is a short summary and outlook.

## II. MOLECULAR STATE QCD SUM RULES

The starting point of the QCD sum rule method is to construct the interpolating current properly and then write down the correlator (for reviews see [21–24] and references therein). The molecular state currents are built up with the color-singlet currents for their composed hadrons. As for  $B^*\bar{B}$ , the current is constructed as

$$j_{B^*\bar{B}}^\mu = (\bar{q}_e \gamma^\mu b_e)(\bar{b}_f i\gamma_5 q_f), \quad (1)$$

where  $q$  indicates the light quark and the subscript  $e$  and  $f$  are color indices. Note that the current is local and the four field operators act at the same space-time point. It is a limitation inherent in the QCDSR disposal of the hadrons since the bound states are not point particles in a rigorous manner. The current is different from that of tetraquark state which is diquark-antidiquark configuration of fields. These two types of currents can be related to each other by Fiertz rearrangement and differ by color and Dirac factors [25]. It will have a maximum overlap for the molecular state with the meson-meson type current. The masses of the  $b\bar{b}q\bar{q}$  tetraquark states were calculated in QCDSR and found to be around  $10.1 \sim 10.3$  GeV in [19], and  $10144 \pm 106$  MeV in [20]. Both of the predicted values for tetraquark states are lower than the  $Z_b$  mass. Therefore, it's hard to accommodate  $Z_b$  as tetraquark states.

The two-point correlator is defined as

$$\Pi^{\mu\nu}(q^2) = i \int d^4x e^{iq \cdot x} \langle 0 | T[j_{B^*\bar{B}}^\mu(x) j_{B^*\bar{B}}^{\nu+}(0)] | 0 \rangle. \quad (2)$$

Lorentz covariance implies that it can be generally parameterized as

$$\Pi^{\mu\nu}(q^2) = (\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu}) \Pi^{(1)}(q^2) + \frac{q^\mu q^\nu}{q^2} \Pi^{(0)}(q^2). \quad (3)$$

The term proportional to  $g_{\mu\nu}$  will be chosen to extract the mass sum rule. In phenomenology,  $\Pi^{(1)}(q^2)$  can be expressed as

$$\Pi^{(1)}(q^2) = \frac{[\lambda^{(1)}]^2}{M_{B^*\bar{B}}^2 - q^2} + \frac{1}{\pi} \int_{s_0}^{\infty} ds \frac{\text{Im}\Pi^{(1)\text{phen}}(s)}{s - q^2}, \quad (4)$$

where  $M_{B^*\bar{B}}$  denotes the mass of the  $B^*\bar{B}$  resonance,  $s_0$  is the threshold parameter, and  $\lambda^{(1)}$  gives the coupling of the current to the hadron  $\langle 0 | j_{B^*\bar{B}}^\mu | B^*\bar{B} \rangle = \lambda^{(1)} \epsilon^\mu$ . In the OPE side,  $\Pi^{(1)}(q^2)$  can be written as

$$\Pi^{(1)}(q^2) = \int_{4m_b^2}^{\infty} ds \frac{\rho^{\text{OPE}}(s)}{s - q^2}, \quad (5)$$

where the spectral density is  $\rho^{\text{OPE}}(s) = \frac{1}{\pi} \text{Im}\Pi^{(1)}(s)$ . Applying quark-hadron duality and making a Borel transform, we have the sum rule from Eqs. (4) and (5)

$$[\lambda^{(1)}]^2 e^{-M_{B^*\bar{B}}^2/M^2} = \int_{4m_b^2}^{s_0} ds \rho^{\text{OPE}}(s) e^{-s/M^2}, \quad (6)$$

with  $M^2$  the Borel parameter. Making the derivative in terms of  $M^2$  to the sum rule and then dividing by itself, we have the mass of the  $B^*\bar{B}$  state

$$M_{B^*\bar{B}}^2 = \int_{4m_b^2}^{s_0} ds \rho^{\text{OPE}} s e^{-s/M^2} / \int_{4m_b^2}^{s_0} ds \rho^{\text{OPE}} e^{-s/M^2}. \quad (7)$$

For the OPE calculations, we work at leading order in  $\alpha_s$  and considers condensates up to dimension six, with the similar techniques developed in [26]. To keep the heavy-quark mass finite, the momentum-space

expression for the heavy-quark propagator and the expressions with two and three gluons attached are used [27]. The light-quark part of the correlation function is calculated in the coordinate space and then Fourier-transformed to the momentum space in  $D$  dimension. The resulting light-quark part is combined with the heavy-quark part before it is dimensionally regularized at  $D = 4$ . The spectral density can be written as

$$\rho^{\text{OPE}}(s) = \rho^{\text{pert}}(s) + \rho^{\langle \bar{q}q \rangle}(s) + \rho^{\langle \bar{q}q \rangle^2}(s) + \rho^{\langle g\bar{q}\sigma \cdot Gq \rangle}(s) + \rho^{\langle g^2 G^2 \rangle}(s) + \rho^{\langle g^3 G^3 \rangle}(s), \quad (8)$$

where  $\rho^{\text{pert}}$ ,  $\rho^{\langle \bar{q}q \rangle}$ ,  $\rho^{\langle \bar{q}q \rangle^2}$ ,  $\rho^{\langle g\bar{q}\sigma \cdot Gq \rangle}$ ,  $\rho^{\langle g^2 G^2 \rangle}$ , and  $\rho^{\langle g^3 G^3 \rangle}$  are the perturbative, quark condensate, four-quark condensate, mixed condensate, two-gluon condensate, and three-gluon condensate spectral densities, respectively. They are

$$\begin{aligned} \rho^{\text{pert}}(s) &= \frac{3}{2^{12}\pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^3} (1-\alpha-\beta)(1+\alpha+\beta)r(m_b, s)^4, \\ \rho^{\langle \bar{q}q \rangle}(s) &= -\frac{3\langle \bar{q}q \rangle}{2^7\pi^4} m_b \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^2} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta} (1+\alpha+\beta)r(m_b, s)^2, \\ \rho^{\langle \bar{q}q \rangle^2}(s) &= \kappa \frac{\langle \bar{q}q \rangle^2}{2^4\pi^2} m_b^2 \sqrt{1-4m_b^2/s}, \\ \rho^{\langle g\bar{q}\sigma \cdot Gq \rangle}(s) &= \frac{3\langle g\bar{q}\sigma \cdot Gq \rangle}{2^8\pi^4} m_b \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \left\{ \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta} r(m_b, s) - \frac{2}{1-\alpha} [m_b^2 - \alpha(1-\alpha)s] \right\}, \\ \rho^{\langle g^2 G^2 \rangle}(s) &= \frac{\langle g^2 G^2 \rangle}{2^{11}\pi^6} m_b^2 \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\min}}^{1-\alpha} d\beta (1-\alpha-\beta)(1+\alpha+\beta)r(m_b, s), \\ \rho^{\langle g^3 G^3 \rangle}(s) &= \frac{\langle g^3 G^3 \rangle}{2^{13}\pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\min}}^{1-\alpha} d\beta (1-\alpha-\beta)(1+\alpha+\beta)[r(m_b, s) + 2m_b^2\beta], \end{aligned}$$

with  $r(m_b, s) = (\alpha + \beta)m_b^2 - \alpha\beta s$ . The integration limits are given by  $\alpha_{\min} = (1 - \sqrt{1 - 4m_b^2/s})/2$ ,  $\alpha_{\max} = (1 + \sqrt{1 - 4m_b^2/s})/2$  and  $\beta_{\min} = \alpha m_b^2 / (s\alpha - m_b^2)$ . We have applied  $\langle q\bar{q}q\bar{q} \rangle = \kappa \langle q\bar{q} \rangle^2$  to estimate the four-quark condensate, where the parameter  $\kappa$  is introduced to account for deviation from the factorization hypothesis [22].

### III. NUMERICAL ANALYSIS

In this section, the Eq.(7) will be numerically analyzed. The  $b$  quark mass is taken as  $m_b = 4.20_{-0.07}^{+0.17}$  GeV [28]. The condensates are  $\langle \bar{q}q \rangle = -(0.23 \pm 0.03)^3$  GeV<sup>3</sup>,  $\langle g\bar{q}\sigma \cdot Gq \rangle = m_0^2 \langle \bar{q}q \rangle$ ,  $m_0^2 = 0.8$  GeV<sup>2</sup>,  $\langle g^2 G^2 \rangle = 0.88$  GeV<sup>4</sup>, and  $\langle g^3 G^3 \rangle = 0.045$  GeV<sup>6</sup> [22, 26]. In the QCDSR approach, there is approximation in the OPE of the correlation function, and there is a very complicated and largely unknown structure of the hadronic dispersion integral in the phenomenological side. Therefore, the match of the two sides is not independent of  $M^2$ . One expects that there exists a range of  $M^2$ , in which the two sides have a good overlap and the sum rule can work well. In practice, one can analyse the convergence in the OPE side and the pole contribution dominance in the phenomenological side to determine the allowed Borel window: on one hand, the lower constraint for  $M^2$  is obtained by the consideration that the perturbative contributions should be larger than the condensate contributions, so that the convergence of the OPE is under control and the higher dimension terms can be safely ignored; on the other hand, the upper limit for  $M^2$  is obtained by the restriction that the pole contributions should be larger than the continuum state contributions, so as to guarantee that the contributions from high resonance states and continuum states remains a small part in the phenomenological side. Meanwhile, the threshold parameter  $\sqrt{s_0}$  is not completely arbitrary but characterizes the beginning of the continuum states. The energy gap between the groundstate and the first excitation is around 500 MeV in many cases of nucleons or charmonium-like states. Whereas,

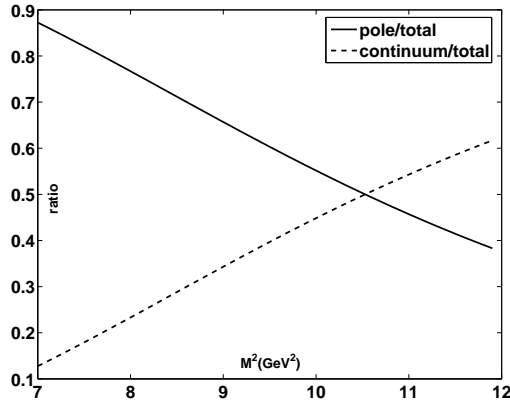


FIG. 1: The phenomenological contribution in sum rule (6) for  $\sqrt{s_0} = 11.4$  GeV. The solid line is the relative pole contribution (the pole contribution divided by the total, pole plus continuum contribution) as a function of  $M^2$  and the dashed line is the relative continuum contribution.

this may not be always straightforwardly generalized to the bottomonium-like states since their absolute masses are much larger than masses of nucleons or charmonium-like states. One should consider whether there is a proper region of Borel parameter  $M^2$  for a fixing  $\sqrt{s_0}$ , or whether one could find a working window for the sum rule. On all accounts, it is expected that the two sides have a good overlap in the determined work window and information on the resonance can be safely extracted.

Firstly, we take the factorization hypothesis of the four-quark condensate  $\langle q\bar{q}q\bar{q} \rangle = \kappa \langle q\bar{q} \rangle^2$  with  $\kappa = 1$ . The comparison between pole and continuum contributions of sum rule (6) as a function of the Borel parameter  $M^2$  for the threshold value  $\sqrt{s_0} = 11.4$  GeV is shown in FIG. 1. Its OPE convergence by comparing the perturbative, quark condensate, four-quark condensate, mixed condensate, two-gluon condensate, and three-gluon condensate contributions as a function of  $M^2$  is shown in FIG. 2. The ratio of perturbative contributions to the total OPE contributions at  $M^2 = 9.0$  GeV<sup>2</sup> is nearly 58% and it increases with  $M^2$ . Thus the perturbative contributions will dominate in the total OPE contributions when  $M^2 \geq 9.0$  GeV<sup>2</sup>. On the other hand, the relative pole contribution is approximate to 50% at  $M^2 = 10.5$  GeV<sup>2</sup> and it decreases with  $M^2$ . In order to guarantee that the pole contribution can dominate in the total contributions, we have the value  $M^2 \leq 10.5$  GeV<sup>2</sup>. Thus, the range of  $M^2$  for  $B^*\bar{B}$  is taken as  $M^2 = 9.0 \sim 10.5$  GeV<sup>2</sup> for  $\sqrt{s_0} = 11.4$  GeV. Similarly, the proper range of  $M^2$  is  $9.0 \sim 9.8$  GeV<sup>2</sup> for  $\sqrt{s_0} = 11.2$  GeV, and  $9.0 \sim 11.2$  GeV<sup>2</sup> for  $\sqrt{s_0} = 11.6$  GeV. We see that the corresponding Borel parameter range is  $M^2 = 9.0 \sim 9.1$  GeV<sup>2</sup> for  $\sqrt{s_0} = 11.0$  GeV, and  $M^2 = 9.0 \sim 9.4$  GeV<sup>2</sup> for  $\sqrt{s_0} = 11.1$  GeV, which are very narrow as working windows. Therefore, the threshold parameter  $\sqrt{s_0}$  is taken as  $11.2 \sim 11.6$  GeV. The mass of  $B^*\bar{B}$  is numerically calculated to be  $10.56 \pm 0.18$  GeV and shown in FIG. 3.

The coupling constant  $\lambda^{(1)}$  between current and the particle is calculated from Eq.(6) in the same working windows. We arrive at  $\lambda^{(1)} = 0.27 \pm 0.07$  GeV<sup>5</sup>. The equivalent quantity of the tetraquark current is  $0.09 \sim 0.11$  GeV<sup>5</sup> in [20]. The coupling constant  $\lambda^{(1)}$  is roughly three times as large as the one of the tetraquark current and the meson-meson molecular current has a larger overlap with the  $Z_b$  state in comparison to the diquark-antidiquark tetraquark current.

To investigate the effect of the factorization breaking, we assume that  $\langle q\bar{q}q\bar{q} \rangle = \kappa \langle q\bar{q} \rangle^2$  with  $\kappa = 2$ . From the similar analysis process, the corresponding working windows are taken as:  $M^2 = 9.0 \sim 9.9$  GeV<sup>2</sup> for  $\sqrt{s_0} = 11.2$  GeV,  $M^2 = 9.0 \sim 10.6$  GeV<sup>2</sup> for  $\sqrt{s_0} = 11.4$  GeV, and  $M^2 = 9.0 \sim 11.4$  GeV<sup>2</sup> for  $\sqrt{s_0} = 11.6$  GeV. We extract the mass value  $10.52 \pm 0.21$  GeV. Finally, we average two results for  $\kappa = 1, 2$  and arrive at the mass value  $10.54 \pm 0.22$  GeV for  $B^*\bar{B}$ , which agrees with the experimental value  $10608.4 \pm 2.0$  MeV for  $Z_b(10610)$ .

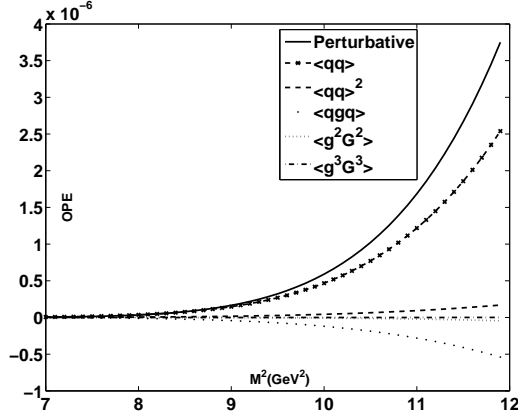


FIG. 2: The OPE contribution in sum rule (6) for  $\sqrt{s_0} = 11.4$  GeV. The OPE convergence is shown by comparing the perturbative, quark condensate, four-quark condensate, mixed condensate, two-gluon condensate, and three-gluon condensate contributions.

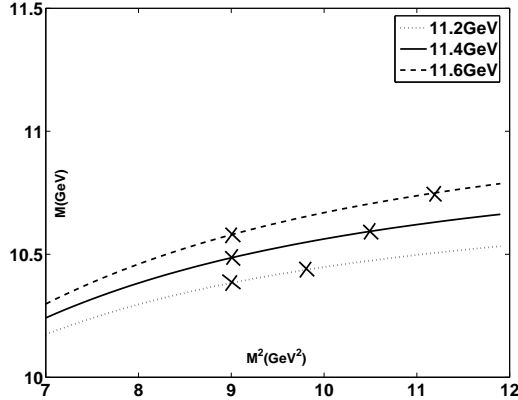


FIG. 3: The mass of the  $B^* \bar{B}$  molecular state as a function of  $M^2$  from sum rule (7). The continuum thresholds are taken as  $\sqrt{s_0} = 11.2 \sim 11.6$  GeV. The ranges of  $M^2$  is  $9.0 \sim 9.8$   $\text{GeV}^2$  for  $\sqrt{s_0} = 11.2$  GeV,  $9.0 \sim 10.5$   $\text{GeV}^2$  for  $\sqrt{s_0} = 11.4$  GeV, and  $9.0 \sim 11.2$   $\text{GeV}^2$  for  $\sqrt{s_0} = 11.6$  GeV.

#### IV. SUMMARY AND OUTLOOK

By assuming  $Z_b(10610)$  as a  $B^* \bar{B}$  molecular state, the QCD sum rule method has been applied to calculate the mass of the resonance. Our numerical result is  $10.54 \pm 0.22$  GeV for  $B^* \bar{B}$ . It is compatible with the newly measured experimental data of  $Z_b(10610)$  by Belle Collaboration, which supports the statement that  $Z_b(10610)$  resonance could be a  $B^* \bar{B}$  molecular state. It is expected that this work is helpful for understanding the structure of  $Z_b(10610)$ . For the newly observed structure  $Z_b(10650)$ , one could consider how to construct a  $B^* \bar{B}^*$  molecular state current with a quantum number of  $J^P = 1^+$  from the meson-meson configuration of fields. For further work, one needs to take into account other dynamical analysis to identify the structures of  $Z_b$  hadrons.

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